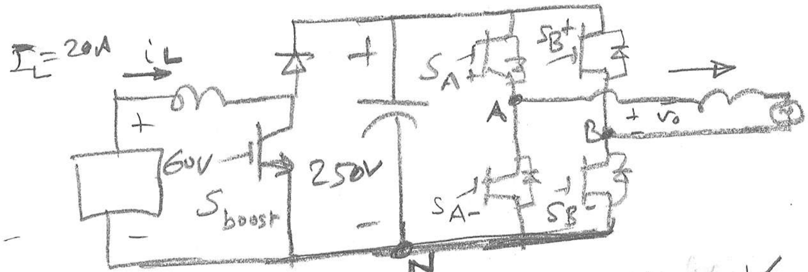
Problem 9b (2 points)

A Photovoltaic panel under a given insolation condition operates at its maximum power point with the voltage of 60 V and the current of 20 A. This PV panel supplies power to a 120-V, 60-Hz, residential grid. The PV interface consists of a boost dc-dc converter, followed by a full-bridge, single-phase, dc-ac inverter. Assume all components to be ideal (i.e., zero power loss everywhere for the sake of simplicity).

The output of the boost dc-dc converter, operating in a continuous-current-conduction mode at a switching frequency of 250 kHz, is 250 V (dc) across a very large dc-link capacitor such that the ripple in the capacitor voltage can be assumed to be negligible. This capacitor voltage acts as the input to a single-phase, full-bridge, inverter that supplies power to the grid at a unity power factor; assume the ripple in the output current to be negligible. Inverter switches are sine-PWM controlled with the switching frequency of 100 kHz. Assume the inductance on the ac-side of the inverter to be negligible.

(a) Draw the circuit diagram of such an interface and label all the components.
(b) What is the duty-ratio of the transistor-switch in the boost dc-dc converter?
(c) In the single-phase inverter, over one switching time-period, plot the duty-ratios, each below the other, of all the four transistor-switches as functions of time when the grid voltage is at its positive peak. Label your plots with appropriate numerical values.
(d) What is the peak value of the current into the residential grid?

(a)



(b) $\frac{V_o}{V_{in}} = \frac{1}{1 - D_{boost}}$ $\therefore V_o - D_{boost} V_o = V_{in}$

$D_{boost} = \frac{V_o - V_{in}}{V_o} = 1 - \frac{V_{in}}{V_o} = 1 - \frac{60}{250} = 0.76$

(c) $\bar{v}_o = \sqrt{2} \times 120 \sin \omega t = 169.7 \sin \omega t$

$\bar{v}_{AN} = \frac{V_{dc}}{2} + \frac{\bar{v}_o}{2}$, $\bar{v}_{BN} = \frac{V_{dc}}{2} - \frac{\bar{v}_o}{2}$

$d_{SA+} = \frac{\bar{v}_{AN}}{V_{dc}} = \frac{1}{2} + \frac{1}{2} \frac{\bar{v}_o}{V_{dc}} = \frac{1}{2} + \frac{1}{2} \frac{169.7}{250} = 0.839$

$d_{SA-} = 1 - d_{SA+} = 1 - 0.84 = 0.161$

$d_{SB+} = \frac{\bar{v}_{BN}}{V_{dc}} = \frac{1}{2} - \frac{1}{2} \frac{\bar{v}_o}{V_{dc}} = 0.161$

$d_{SB-} = 1 - d_{SB+} = 0.839$

(d) $P = 60 \times 20 = 1,200 \text{ W}$

$I_{peak} = \sqrt{2} \frac{1200}{120} = 14.14 \text{ A}$

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Solution

Problem 9 (b) A synchronous machine is connected to an infinite bus as shown in the figure below. The machine has the following parameters: synchronous speed, $\omega_s = 120\pi$ [rad/s], inertia constant, $H = 8$ [s], machine damping, $D = 0.004$ [rad^{-1}s], machine terminal voltage, $E = 0.9$ [pu], mechanical power input, $P_M = 1.0$ [pu], machine terminal impedance, $X_M = 0.2$ [pu], the infinite bus voltage magnitude, $v_\infty = 1.0$ [pu], and the line impedance, $X_L = 0.25$ [pu]. Recall that the *swing equations* that govern the evolution of the rotor angle, δ, and frequency, ω, are given by:

$$\frac{d\delta}{dt} = \omega - \omega_s,$$
$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = P_M - P_E - D(\omega - \omega_s), \tag{1}$$

where P_E is the electrical output power sourced by the generator from terminal 1.

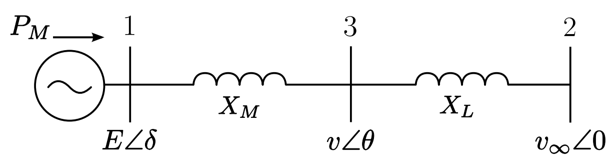

Figure 1: Single line diagram of single machine infinite bus

Solution 9 (b)

(i) Write an expression for the electrical output power, P_E. Your expression should only be a function of E, v_∞, δ, X_M, and X_L.
Solution:

$$P_E = \frac{Ev_\infty}{X_M + X_L}\sin\delta \tag{2}$$

(ii) Find the equilibrium points for the system in (1) in the interval, $[0, \pi]$.
Solution:
Swing equations are given by:

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} + D\frac{d\delta}{dt} = P_M - P_E,$$
$$\rightarrow \quad 0.0424\frac{d^2\delta}{dt^2} + 0.004\frac{d\delta}{dt} = 1 - 2\sin\delta$$

$$1 - 2\sin\bar{\delta} = 0$$
$$\rightarrow \quad \bar{\delta} = \frac{\pi}{6}, \frac{5\pi}{6}$$

(iii) Linearizing the system around the equilibrium point, find the eigenvalues.
Solution:

$$x_1 = \delta, \quad x_2 = \frac{d\delta}{dt}$$

Then,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{0.0424}\left(1 - 2\sin x_1 - 0.004 x_2\right)\end{bmatrix}$$

Linearization around the equilibrium point, $\bar{x}_1$, is:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} \simeq \begin{bmatrix} 0 & 1 \\ -\frac{2}{0.0424}\cos x_1 & -0.004\end{bmatrix}\begin{bmatrix} x_1 \\ x_2\end{bmatrix}$$

Linearization around the equilibrium points, $\bar{x}_1 := \bar{\delta} = \frac{\pi}{6}$, yields:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} \simeq \begin{bmatrix} 0 & 1 \\ -40.8503 & -0.0943\end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2\end{bmatrix}$$

Linearization around the equilibrium points, $\bar{x}_1 := \bar{\delta} = \frac{5\pi}{6}$, yields:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} \simeq \begin{bmatrix} 0 & 1 \\ 40.8503 & -0.0943\end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2\end{bmatrix}$$

(iv) Mention which equilibrium point is stable.

Solution:

Eigenvalues for the equilibrium point, $\bar{x}_1 := \bar{\delta} = \frac{\pi}{6}$, are $\lambda_{1,2} = -0.0471 \pm \mathrm{j}6.3913$, which is stable. Eigenvalues for the equilibrium point, $\bar{x}_1 := \bar{\delta} = \frac{5\pi}{6}$, are $\lambda_{1,2} = 6.3444, -6.4387$, which is unstable.